**8227**
**Paper 2**

**General Comments:**

The paper was of a good standard and fit for the level. The marks ranged between 0 and 75. It was clearly noticed that a significant number of the candidates who entered this syllabus without a solid algebra basis, struggled to cope with the majority of topics.

It should be noted that the emphasis in AS is more on the **method** and the mathematical process than on the final solution. **Complete methods** should be shown at all times.

The presentation of the papers was of a high standard. Candidates must make sure to **delete** work that they do not want to be marked at the specific question.

Candidates should be discouraged to work in pencil first and then to rewrite their work in pen. They often do not copy the pencil properly and some accuracy is lost after the pencil is erased. Centres must **NOT** provide any **additional paper**. Candidates do their work on these papers and then just copy the answers, which results in the loss of all the marks for the specific question.

It is advised that answers should be left **exact** where possible. An answer of $\sqrt{18}$ or $3\sqrt{2}$ is exact and is preferred above the decimal version of 4.2426… Candidates should be encouraged only to round their **final** answer. This prevents the loss of accuracy caused by premature rounding, especially in the Trigonometry questions.

The lack of basic arithmetic skills was sadly observed throughout the paper.

It was noted that some candidates used **calculators** that can **differentiate and integrate**. Candidates who use these, must be taught to show **all the steps** of their working. **No marks** are awarded if only answers are given.

Most candidates took care to show their working, which allows them to gain marks for correct methods even when their answers are wrong. However, candidates should be encouraged to show complete methods, especially when they are asked to prove a **quoted** answer. It should be emphasised that "**SHOWN**" is **NOT** an acceptable conclusion. Candidates may **NOT** work **backwards** from the quoted answer.

Candidates should realise that questions are structured in an attempt to help them. The earlier parts of a question are often used in subsequent parts. The word "**HENCE**" is an indication that the previous answer must be used in order to solve the specific question. Candidates **must** be taught how to interpret questions of this nature. If the question states "**hence**" and the previous question is not used, no marks may be awarded for the solution. It should be noted that none of the questions in this paper read "…**hence, or otherwise**, …." If the "…. or otherwise…." is seen it is an indication that the previous answer should assist the candidate to solve this question easier, but there are other (more complicated) methods that could be used.

Most candidates finished the paper. It was clear from the candidate's work that some centres did not complete the syllabus for theme 2.

Some candidates tend to write the same solution in different forms. This should not be encouraged, as wrong working could spoil perfectly correct methods and answers. There is no need to write $\frac{3}{4}e^{-3x}$ or any other exponents with **positive indices** in the final solution.

Candidates must not be encouraged to use **tables and plotting points** to enable them to draw basic graphs. The basic shapes of the graphs, as well as their **transformations must be taught**.

**Comments on specific questions:**

**Question 1**

**(a)** Very few candidates did not realise that they had to use the remainder theorem. Most candidate who did not score full marks for this question, lost the last mark that was for solving the simultaneous equations correctly.

**(b)** It was clear that many teachers teach the use of tables of the calculator to find the first factor. If tables are used, it is advisable to search for **integer factors**. If was often seen that $x = \frac{1}{2}$ was used. It should be noted that the factor will then be $(2x – 1)$ and not $\left(x - \frac{1}{2}\right)$. If synthetic division is used with the $\frac{1}{2}$, the candidates must be taught to divide the answer of $6x^2 – 2x – 4$ by 2 to find the **correct quotient** of $3x^2 – x – 2 = (3x + 2)(x – 1)$. Candidates often failed to find the 3 solutions after the 3 factors were found. This resulted in the loss of the last accuracy mark. Some candidates also omitted the solution of the first factor, mostly $x = 1$ in the final solution.

It was also regularly seen that candidates did not realise that they had to use the factor theorem and randomly tried to factorise and solve an equation, which was often equal to – 2.

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| **1 (a)** | $4m – 2n = 56$<br><br>$4m + 2n = 0$<br><br>$m = 7 \quad n = – 14$ | M 1<br><br>M1<br><br>A 1 | Substitute in $x$ = - 2 to obtain $-16+4m-2n+5=45$, allow one slip.<br><br>Substitute in $x$ = +2 to obtain $16+4m+2n+5=21$, allow one slip. | |
| **1 (b)** | $x – 1$<br><br>$6x^2 + x – 2$<br>$(x – 1)(2x – 1)(3x + 2)$<br>$x = 1$ or $x =$ or $x =$ | M 1<br><br>A 1<br>DM 1<br>A 1 | Finding first factor together with division with 2 terms correct.<br>For the correct quadratic<br>Solving quadratic<br>Answers with no working, only 0/4 | **7** |

**Question 2**

The answer to this question was quoted and it was expected of candidates to integrate $\frac{8}{2x+1}$ to find $\frac{8\ln(2x+1)}{2}$ to which the two boundaries 4 and 1 had to be applied correctly. It was regularly seen that candidates did not put the $(2x + 1)$ in brackets and then could not deal with the boundaries properly. As the answer was quoted, candidates were expected to show all their algebraic working. 4ln 9 – 4ln 3 could not be written to ln 81 without using any logarithmic laws. 4ln 9 – 4ln 3 could also not be equated to ln 81 by working out the decimal version 4.39444… and then conclude that the two are equal. This was a definite integral, so there is no use to add the constant (c).

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| **2** | $\frac{8}{2}\ln(2x + 1)$<br><br>$4\ln(8 + 1) – 4\ln(2 + 1)$<br><br>$= 4 \ln 3$<br>$= \ln 81$ | B 1<br><br>B1<br><br>M 1<br><br>E 1 | B1for $k\ln(2x + 1)$<br><br>B1for $\frac{8}{2}\ln(2x + 1)$<br><br>For applying limits correctly to an integral of the form $k\ln(2x+1)$<br><br>**Use log laws and establish the given result** | **4** |

**Question 3**

Most candidates could use a correct method to calculate the two critical values (1 and – 2).
From this point onwards, the minority of the candidates succeeded to conclude with the correct **DOUBLE INEQUALITY**. It should be noted that it is one region and the final answer must be one inequality and not two separate inequalities.

A very common mistake seen was $3x + 3 = -x + 5$ instead of $3x + 3 = -x - 5$. This resulted in the loss of the second method mark, as well as both accuracy marks.

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| **3** | $9x^2 + 18x + 9 < x^2 + 10x + 25$<br><br>$8x^2 + 8x - 16 < 0$ or $= 0$<br>$(x + 2)(x - 1) < 0$ or $= 0$<br>$-2 < x < 1$<br><br>OR<br><br>Consider one linear equation or inequality to obtain 1 critical value<br>Consider another linear equation or inequality to obtain another critical value<br><br>Obtain the critical values 1 and – 2<br><br>$-2 < x < 1$ | M 1<br><br>A 1<br>M 1<br>A 1<br><br><br><br>M1<br><br>M1<br><br>A1<br>A1 | For squaring both sides (condone inclusion of = rather than <)<br><br>Obtain two critical values<br>Must be a single inequality<br><br><br><br>$3x+3 <,\ >,\ \text{or}\ = x+5$<br><br>$3x+3 <,\ >,\ \text{or}\ = -x-5$<br><br><br>**Must be a single inequality** | **4** |

**Question 4**

**(a)** The minority of candidates scored any marks in this question. Candidates must be taught to study the basic shapes of the exponential and logarithmic graphs, as well as their transformations. Most candidates tried to use the table function in the calculator and ended up with a shape that is not a smooth descending exponential curve, with the $x$-axis as asymptote. It should also be noted that the time starts at t = 0. The descending curve must only be in the first quadrant.

**(b)** It was occasionally seen that candidates just tried out different numbers ("trial and error method") and then concluded to a number of weeks. It is not an acceptable method for Advanced Subsidiary Level. It was expected of candidates to be able to deal with the appropriate algebra. The question asked for the number of complete weeks. The expectation was to first see the decimal version of exactly the half of 5000, followed by a conclusion.

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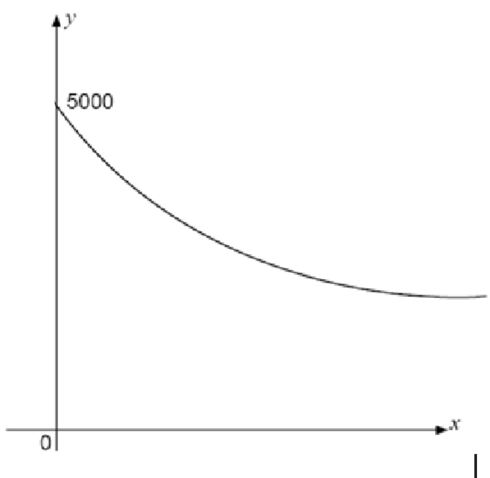
| **4 (a)** |  | B 1<br><br>B 1 | Correct shape in the first quadrant only<br><br>5000 seen with a correct shape (allow if in 2nd quadrant) | |
| **4 (b)** | $2500 = 5000e^{-0.02t}$<br>$-0.02t = \ln 0.5$<br>$t = 34.657$<br><br>$t = 34$ | B 1<br>M 1<br><br>A 1 | For a complete method to find $t$, condone use of inequalities<br><br>Accept: 35 | **5** |

**Question 5**

**(a)** Most candidates realised that the derivative of ln $x$ is $\frac{1}{x}$. It was sadly very often seen that candidates thought that $\ln x = \frac{1}{x}$. Very few candidates realised that they had to use the product rule to differentiate $x$ ln $x$. Some candidates wrongly attempted to apply logarithmic laws to the expression. Another common mistake was to apply brackets around

ln($x - x$). Candidates did not realise that they changed the whole expression by adding brackets.

Sadly, it was often seen that $\frac{x}{x} + \ln x - 1$ was not simplified to ln $x$. This resulted in the loss of the final accuracy mark.

**(b)** This question states "Hence evaluate….". Candidates were expected to realise that an integral is an anti-derivative. They were expected to apply the two boundaries to the given expression in **part (a)**.

The correct answer was often seen from incorrect working or no working at all. This meant that the candidate used a calculator that can evaluate definite integrals. These candidates did not score any marks. Complete methods must be shown at all times.

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| **5 (a)** | $x\frac{d}{dx}(\ln x) + (\ln x)\frac{d}{dx}(x) - 1$<br>$= x \times \frac{1}{x} + \ln x - 1$<br>$= \ln x$ | M 1<br>B 1<br>A 1 | Use of product rule, allow one slip<br>$\frac{d}{dx}(\ln x) = \frac{1}{x}$ seen | |
| **5 (b)** | $x \ln x - x$<br>$2 \ln 2 - 2 - (1 \ln 1 - 1)$<br>$= 2 \ln 2 - 1$ or $\ln 4 - 1$<br>$(= 0.386)$ | B 1<br><br>B 1 | **Answer only 0/2** | **5** |

**Question 6**

**(a)** Most candidates could use the two formulae correctly and then replace the appropriate trigonometric ratios. After the initial two marks, most candidates failed to score the final two marks, due to very poor arithmetic skills. Once again, it was a quoted answer and no steps may be skipped in the process leading to the quoted answer.

**(b)** Many candidates, again, did not realise that they needed to use the quoted answer to the previous question. Those that realised that they had to do that, often sadly did not know that they needed to write the equation to tan $x$. It was seen regularly that candidates correctly concluded that $\cot x = \sqrt{3}$, but failed to convert that correctly to $\tan x = \frac{1}{\sqrt{3}}$ in order to find the solution of 30°. A common wrong answer given was 60°. An acute angle was asked. Candidates who gave 2 solutions instead of one, lost the accuracy mark.

The question was given in degrees, so the solution had to be in degrees.

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| **6 (a)** | $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = 2\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right)$<br>$= \sqrt{3}\cos x - \sin x$<br>$\frac{3}{2}\sin x = \frac{\sqrt{3}}{2}\cos x$<br>$\sqrt{3}\sin x = \cos x$ | M 1<br>M 1<br>M 1<br>E 1 | Use of the correct formula<br>Correct substitution of trig ratios, allow one slip<br>Collect like terms | |
| **6 (b)** | $\tan x = \frac{1}{\sqrt{3}}$<br>$x = 30°$ | M 1<br>A 1 | Using $\tan x = \frac{\sin x}{\cos x}$ = positive ratio<br>A0 if more than 1 solution | **6** |

**Question 7**

Most candidates could correctly differentiate the parametric equations. Very few candidates realised that they needed to equate $y$ = 0 in order to find the t-values where the curve meets the $x$-axis.  They then needed to substitute these t values to find the appropriate $x$-values.

Sadly, many candidates concluded that $x$ = 0 will give the points where the curve meets the $x$-axis. Very few candidates scored more than the two marks for the differentiation for this question.

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| 7 | $t = 0, t = -1$<br>$x = 0$ or $x = 20$<br>$m = \frac{dy}{dx} = \frac{1+2t}{4t-18}$<br><br>$m = -\frac{1}{18}$  $m = \frac{1}{22}$<br><br>$y = -\frac{1}{18}x$<br><br>$y = \frac{1}{22}(x-20)$ or $y = \frac{x}{22} - \frac{10}{11}$ | M 1<br><br>M 1<br><br>A 1<br><br>DM 1<br><br>A 1<br><br>A 1 | Solving $y = 0$ and attempt to find corresponding $x$ values<br>Attempt differentiation of all terms to obtain $\frac{dy}{dt} \div \frac{dx}{dt}$, must have either denominator or numerator correct<br>All correct<br><br>**Dependent on both previous M marks.**<br>Find the gradient and the equation of one tangent. | **6** |

**Question 8**

**(a)** In general, this question was answered very well. It was occasionally seen that candidates failed to conclude after they found the two values 13 and – 2. It was expected that they refer to "change of sign" in order to gain the mark for the conclusion.

**(b)** Most candidates answered this question successfully and scored both marks. It was occasionally seen that candidates attempted to work backwards from the answer, which resulted in the loss of all the marks.

**(c) (i)** Some candidates failed to use the given value $x$ = 2 as a starting value.  These candidates lost all the marks for this question. Other candidates were confused with the given notation. Throughout, the first two solutions the candidates wrote down were marked.  It was also seen that the required accuracy of 5 d.p. was not adhered to.  No marks were awarded for accuracy less than 5 d.p..

**(c) (ii)** Many candidates did not realise that they had to continue with the iteration process they started in the previous question to find the solution to the given iterative formula.  Some candidates found the correct solution, but did not realise that was the answer to the question.  They substituted it into the given formula.  If the correct solution was seen in the substitution, candidates were awarded the mark.

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| **8 (a)** | $(2)^3 - 4(2) - 2 = -2$<br>$(3)^3 - 4(3) - 2 = 13$<br>Change of sign | B 1<br>E 1 | Show both substitutions<br>Any other valid explanation | |
| **8 (b)** | $x^3 = 2 + 4x$ leading to $x^2 = \frac{2}{x} + 4$<br>or equivalent<br>$x = \sqrt{\frac{2}{x} + 4}$ | M 1<br><br>E 1 | For re-arranging to obtain a term in $x^2$ | |
| **8 (c)(i)** | $x_2 = 2.23607$ or $\sqrt{5}$<br>$x_3 = 2.21234$ | B 1<br>B 1 | Allow truncated to 2.23606<br>Allow truncated to 2.21233 | |
| **8c)(ii)** | 2.2143 | B 1 | | **7** |

**Question 9**

**(a)** The minority of the candidates realised that they needed to divide both terms in the numerator by the denominator. Those who realised that, often had difficulty dealing with the index rules correctly.

Sadly, it was often seen that candidates attempted to use the quotient rule, not realising that there is no quotient rule for integration in their syllabus.

This is an indefinite integral and it was expected that they add a constant value **(c)** to the expression. There was no need to leave the final answer with positive indices.

**(b)** Many candidates could not deal with the identity given on the formula sheet correctly. Untidy and careless work was seen in this question. Often $\cos^2 x$ was replaced by $\cos 2x$. Some candidates attempted to use the identity $\sin^2 x + \cos^2 x = 1$, which did not help them to obtain an expression in terms of $\cos 2x$. Once again, this is an indefinite integral and the final accuracy mark was lost for omitting the constant.

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| **9 (a)** | $\int(4e^{-3x} - 2)\mathrm{d}x$<br>$= \frac{4e^{-3x}}{-3} - 2x \quad (+c)$ | M 1<br>A 2, 1 | Dividing both terms by $e^{3x}$<br>1 mark off for each error in a single term | |
| **9 (b)** | Attempt to use $\frac{\cos 2x+1}{2} = \cos^2 x$<br>$\int(2\cos 2x + 2)dx$<br>$= \sin 2x + 2x + c$ | M 1<br>A1<br>A 1<br>A 1 | For obtaining the form $\int(k_1 \cos 2x + k_2)\mathrm{d}x$ where $k_2 \neq 0$<br>All correct<br>$\sin 2x$<br>$2x + c$ | 7 |

**Question 10**

**(a)** Candidates must take more care with the writing of implicit derivates. Many candidates wrote it in the unsimplified form and expected markers to do the differentiation for them.

$\frac{\mathrm{d}}{\mathrm{d}x}3x \times y + 3x\frac{\mathrm{d}}{\mathrm{d}y}y\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\mathrm{d}}{\mathrm{d}y}5y\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\mathrm{d}}{\mathrm{d}x}2x^2 = \frac{\mathrm{d}}{\mathrm{d}x}6$ had to be followed by the next step $3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x} - 5\frac{\mathrm{d}y}{\mathrm{d}x} - 4x = 0$ before the candidate attempted to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject of the formula. It was often seen that candidates wrongly attempted to work backwards from the quoted answer.

**(b)** This question proved to be difficult for the majority of candidates. Most did not realise that they had to use the equation at the top of the question to find the value corresponding to

$x = -1$. This enabled them to find the appropriate gradient of the tangent, using the solution to the previous question. The gradient of the normal is perpendicular to the tangent. The perpendicular gradient had to be used with the calculated coordinate to find the equation of the normal.
Many candidates wrongly assumed the gradient and / or $y = 0$. Many of those who succeeded in calculating a gradient, failed to find the perpendicular gradient before they attempted to find the equation of the normal.

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| **10 (a)** | $3y + 3x\frac{dy}{dx} - 5\frac{dy}{dx} - 4x = 0$<br>$(3x-5)\frac{dy}{dx} = 4x - 3y$<br>$\frac{dy}{dx} = \frac{4x-3y}{3x-5}$ | B 1<br>B 1<br>E 1 | For $3y + 3x\frac{dy}{dx}$<br>For $-5\frac{dy}{dx} - 4x = 0$ | |
| **10 (b)** | $\frac{dy}{dx} = \frac{1}{8}$ , $m_\perp = -8$<br>$y + 1 = -8(x+1)$ or<br>$y = -8x - 9$ | M 1<br>DM 1<br>A 1 | Find gradient and the perpendicular gradient. $(x, y) = (-1, -1)$<br>Find the equation of the normal using *their* perpendicular gradient and calculated $y$ value.<br>Allow equivalents | 6 |

**Question 11**

**(a)** It was shocking how poorly the logarithmic laws were used in this question, considering that logarithms are also part of the Ordinary level (6131) syllabus. Many candidates used the In as a constant that they can expand into the brackets or alternatively that every term can just be divided by a In. A very common mistake seen was that candidates wrote the RHS as $\dfrac{\ln 3x^2}{\ln(2x-1)}$ instead of the correct $\ln\dfrac{3x^2}{2x-1}$.

**(b)** The hence in this question expected the candidates to solve the quadratic equation given in part (a) and then conclude that $x = 2^n$. This enabled them to calculate the 2 expected $n$-values. Most candidates who found the correct $x$-values, failed to continue to find the correct $n$-values. Some of those who succeeded in finding both $n$-values, wrongly concluded that $n = -0.585$ is not a valid solution.

Most candidates attempted to start the question from scratch and showed very poor use of index rules. It was very often seen that $3 \times 2^n$ was wrongly simplified to $6^n$.

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| **11 (a)** | $\ln 3 + \ln x^2 - \ln(2x-1) = \ln 2^2$<br>$\ln\frac{3x^2}{2x-1} = \ln 4$<br>$3x^2 = 4(2x-1)$<br>$3x^2 - 8x + 4 = 0$ | B 1<br>B 1<br><br>E 1 | Use of the power rule twice<br>Correct use of log laws | |
| **11 (b)** | $x = 2$ or $\frac{2}{3}$<br>$n = 1$<br>$2^n = \frac{2}{3}$<br>$n = \log_2\frac{2}{3}$<br>$= -0.585$ | M 1<br>B 1<br><br>M 1<br><br>A 1 | Solve quadratic in **(a)**<br><br><br>Use logs to solve equation | 7 |

**Question 12**

**(a)** This question required that RADIANS must be used. Candidates must be taught to work with radians if the domain is given in radians and with degrees if the domain is given in degrees. It is not advisable that they first do calculations in degrees and then convert the answer to radians. This often resulted in the loss of accuracy due to premature rounding.

**(i)** Sadly, it was very often seen that the angle was calculated as 45°, which resulted in the loss of the accuracy mark. The solutions were expected to be left as $\sqrt{18}$ or $2\sqrt{3}$ and $\frac{1}{4}\pi$ instead of the decimal versions.

**(ii)** Candidates who calculated this solution in degrees lost all the marks in this question. Furthermore, those who used radians, often rounded the value of $\Theta - \frac{1}{4}\pi = 0.123095...$ to 3 s.f. already,

which lead to the loss of accuracy in the final answer. Candidates should be encouraged to always write the solution more accurately and only round the final solution.

**(b)** Many candidates did not realise that they had to use the identity in the formula sheet to convert the double angle tan$2x$ to angles in terms of tan $x$. Those who realised what they had to do, very often failed to do the algebra and arithmetic correctly. Sadly, many candidates concluded that tan $2x$ × tan $x$ = tan$^2 2x$ or a variety of other wrong versions, by presuming that tan$x$ and tan$2x$ are like terms that can be multiplied together.

The domain was given in degrees. Candidates who ignored the given domain and calculated more than the required 2 angles, lost both accuracy marks.

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| **12 (a)(i)** | $R=\sqrt{18}$, $3\sqrt{2}$ or 4.24264…. <br> $\tan\alpha = 1$ <br> $\alpha = 0.785$ or $\frac{\pi}{4}$ | B 1 <br> M 1 <br> A 1 | Any correct method to solve for α | |
| **12 (a)(ii)** | $\theta-\frac{\pi}{4}=1.23095....$ <br> $\theta = 2.02$ radians | M 1 <br> A 1 <br> A 1 | Correct method of solution using part (i) to find $\theta-\frac{\pi}{4}$ = angle in radians <br> For 1.23095… found. <br> One correct value only <br> Allow greater accuracy | |
| **12 (b)** | $4\div\frac{2\tan x}{1-\tan^2 x}$ <br> $4\times\frac{1-\tan^2 x}{2\tan x}$ <br> $3\tan^2 x - 5\tan x - 2 = 0$ o.e. <br> $(3\tan x + 1)(\tan x - 2)$ <br> $\tan x = 2$ <br> $x = 63.4°$ <br> $\tan x = -\frac{1}{3}$ <br> $x = 161.6°$ | M 1 <br> A 1 <br> DM 1 <br> A 1 <br> A 1 | Use identity and deal with $\div\frac{2\tan x}{1-\tan^2 x}$ <br> Simplify to correct quadratic equation <br> Solve quadratic equation <br> A 0 if more solutions given. <br> A 0 if more solutions given <br> Allow greater accuracy | **11** |