MATHEMATICS

8227
Paper 1

Key Messages

Candidates who displayed good algebraic skills coped well with this paper. An alarming number of learners are not up to standard for Paper 1's work: specifically the algebraic execution of the work!
Candidates must be informed that if the question requires to show/prove something, the conclusion must always be stated again.
It is important that candidates understand how to approach the "hence" part of a question.
Calculator and arithmetic skills of many candidates are not up to standard.

General comments

The standard of work in this examination was good for the able candidates, but the weaker candidates' work is not well executed. In general the work was done neatly and clearly. The marks ranged from 0 to 75.
The majority of candidates showed their working. Working can score marks, even if a final answer is wrong.

Although the formulae are given, quite a number of candidates were careless in using them correctly.
The algebraic work was very poor for candidates at this level. In question 3(a) and 11(a) it came forward that candidates do not know that $x^{\text{ even number}}$ x = 81, results into a positive and negative solution for x and $x^{\text{ odd number}}$ x = 8, results into only a positive value for x.

It seems that all candidates were able to finish in time. The weaker candidates left a lot of questions unanswered.

Comments on individual questions

Question 1

Candidates were very careless with their signs of the terms and although the question states non-zero value of p, the 0 was included at times.

1 (a)	$1 - 6px + 15p^2x^2$	B 2,1	1 mark off for each incorrect term
1 (b)	$6px^2 + 15p^2x^2$ $3p(2 + 5p) = 0$ $p = -\frac{2}{5}$	M 1 M 1 A 1	Identify two quadratic terms Solve a two term quadratic equation

Question 2

Many had no idea that the equation of a parabola needs an x^2 term and simply tried with a straight line formula. In general it was well answered.

2	$y = a(x + 2)(x - 4)$ $-3 = a(1 + 2)(1 - 4)$ $a = \frac{1}{3}$ $y = \frac{1}{3}(x + 2)(x - 4)$ OR $y = a(x - 1)^2 - 3$ $0 = a(4 - 1)^2 - 3$ $a = \frac{1}{3}$ $y = \frac{1}{3}(x - 1)^2 - 3$	M 1 M 1 A 1 M 1 M 1 A 1	For $y = a(x + 2)(x - 4)$, allow $a = 1$ For use of $x = 1$ and $y = -3$ Accept: $\frac{1}{3}(x^2 - 2x - 8)$ For use of the minimum point For use of $y = 0$ with either $x = -2$ or $x = 4$ Accept: $\frac{1}{3}(x^2 - 2x - 8)$	**3**

Question 3

The majority of candidates only gave x = 3 and in (b) hence means that the candidate needed to see the relationship with the answers in (a) and not re-calculate the equation.

3 (a)	$x = 3$ $x = -3$	B 1 B 1	
3 (b)	$x = \sqrt{a} - 4$ $a = 1$ or $a = 49$	M 1 B 1 A 1	Realise relationship SC1 if relationship not seen for a = 49 from correct working

Question 4

Some candidates did not realize that $v = \int a\,dt$ and $s = \int v\,dt$ and tried to use formulae from physics.
Many candidates did not find the value of "c" in 4(a) and lost marks in both parts of the question, since they are related.

4 (a)	$v = \int(6t+1)dt$ $v = 3t^2 + t + (c)$ $v = 3t^2 + t + 2$	M 1 A 1 A 1	Attempt to integrate with at least one correct term Correct integration Substitute $v = 3$ and $t = 0$ to find c
4 (b)	$s = \int(3t^2 + t + 2)dt$ $s = t^3 + \frac{1}{2}t^2 + 2t + (c)$ $s = (3)^3 + \frac{1}{2}(3)^2 + 2(3)$ $s = 37.5$ m	M 1 A 1 A 1	Attempt to integrate *their* answer to (a) with at least one correct term Correct integration

Question 5

(a) Well answered, except for those that did not realize that it is a quadratic equation and a lot of candidates could not solve this at this level.

(b) Even with the formulae given, candidates used them wrongly and the execution was done poorly. To proof all steps must be shown to be convincing.

5 (a)	$3\cos^2\theta - 7\cos\theta + 2 = 0$ $\cos\theta = \frac{1}{3}$ or $\cos\theta = 2$ $\theta = 70.5°$ $\theta = 289.5°$	M 1 A 1 B 1 F.T	Multiply throughout by $\cos\theta$ to form and solve a 3 term quadratic equation equated to zero. Allow greater accuracy 360° – "his 70.5°" A0 F.T. if there are more answers	
5 (b)	LHS = $\frac{\sin^2 x}{\cos^2 x} - \sin^2 x$ $= \frac{\sin^2 x - \cos^2 x \sin^2 x}{\cos^2 x}$ $= \frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}$ $= \frac{\sin^2 x}{\cos^2 x} . \sin^2 x$ $= \tan^2 x \ \sin^2 x$ = RHS	B 1 B 1 B 1	Allow working from RHS Use of identity for $\tan^2 x$ and making one fraction Factorising the numerator For correct completion to obtain RHS	**6**

Question 6

Well done in general, but candidates did not find the coordinate of P, some tried to find the unit vector instead. A common wrong way of conclusion was to say "shown", instead of stating that ($\overrightarrow{AP}$) is perpendicular to ($\overrightarrow{OP}$)

6 (a)	$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$ $\overrightarrow{AP} = \begin{pmatrix} -3 \\ 3 \\ -6 \end{pmatrix}$ $\overrightarrow{OP} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$ $P(0, 6, 0)$	M 1 DM 1 M 1 A 1	Complete method to find $\overrightarrow{OP}$
6 (b)	$(-3)(1) + (3)(5) + (-6)(2) = 0$ Conclude	M 1 E 1	For use of scalar product to obtain zero

Question 7

Candidates must adhere to instructions, because many answers were not simplified.
Candidates did not read/understate the description of the metal plate, thinking it was only the shaded region.

7 (a)	$(2\pi - \alpha)r$ $r + r + 2r\alpha$ $r\alpha + 2\pi r + 2r$	B 1 B 1 B 1		
7 (b)	$\frac{1}{2}(2\pi - 0.8)(5)^2$ soi $\frac{1}{2}(10)^2(0.8)$ Final: 109 or $25\pi + 30$	B 1 B 1 B 1	68.54	6

Question 8

Many candidates tried to solve this by using the compound interest formulae and many used AP in (a) and GP in (b).

8 (a)	$r = 1.1 \quad n = 9$ $\frac{1000(1 - 1.1^9)}{1 - 1.1}$ $= 13\ 580$	B 1 M 1 A 1	For r = 1.1 (seen oi) and n = 8 or 9 For use of sum formula for a GP. Allow if done as a sum of 8 or 9 correct separate terms	
8 (b)	$\frac{9}{2}[2(1000) + A(9-1)] = 13579.48$ (or 11435.89) $9000 + 36A = 13579.48$ (or 11435.89) $A = 127.21$	M 1 M 1 A 1	Use sum of A.P with $n = 8$ or 9 Solving of equation seen or implied to give a positive A.	6

Question 9

Well answered. Integration was done well, but wrong limits were applied by some candidates.

9 (a)	$9x - \frac{x^3}{3}$ or $-\frac{2}{3}(9-y)^{\frac{3}{2}}$ $27 - 9$ or $0-(-18)$ $= 18$	M 1 M 1 A 1	Correct integration term or in the form $a(9-y)^{\frac{3}{2}}$ Apply correct limits to an integral $a(9-y)^{\frac{3}{2}}$.	
9 (b)	$\pi\int_0^9 (9-y)dy$ $9y - \frac{y^2}{2}$ $(\pi)(81 - \frac{81}{2} - 0)$ $= \frac{81}{2}\pi$	M 1 A 1 DM 1 A 1	Use of correct formula Apply correct limits, correctly	**7**

Question 10

In (a) candidates were very careless with their signs. When an inequality results into an answer between the two critical values the answer must be written not as two separate inequalities.
In (b) it must be proved firstly and then come to a conclusion. A common wrong conclusion was that $13k^2$ is a perfect square.

10 (a)	$x^2 - 2x + 20 = mx + 4$ $x^2 - mx - 2x + 16 = 0$ $(2 + m)^2 - 4(16)\ (< 0)$ $m^2 + 4m - 60\ (< 0)$ $-10 < m < 6$	M 1 DM 1 DM 1 A 1	Equating to obtain a quadratic equation equated to zero, allow one slip. Finding Δ Solving a quadratic to obtain critical values For the correct inequality	
10 (b)	$k^2 - 4(3)(-k^2)$ $= 13k^2$ $\Delta > 0$, hence 2 real roots	M 1 E 1	For consideration of Δ, must be in terms of k	**6**

Question 11

Well done in general.
In (a) the differentiation was done correctly, but to solve for x seemed to be a problem and $x^3 = 8$ resulted in $x = \pm 2$.
In (b) candidates did not realize that the derivative of (a) was needed as the gradient of the tangent and used $m = \frac{\Delta y}{\Delta x}$ instead.

11 (a)	$y' = 2x - 16x^{-2}$ $2x = \frac{16}{x^2}$ $(x, y) = (2, 12)$	M1 A1 DM1 A1	For attempt at differentiation Correct differentiation Solve equation = 0 Allow $x = 2$ and $y = 12$	
11 (b)	$m = 7 \quad m_\perp = -\frac{1}{7}$ $y - 20 = -\frac{1}{7}(x-4)$ or $\left(y = -\frac{x}{7} + \frac{144}{7}\right)$	M 1 M 1 A 1	Find gradient using calculus and hence perpendicular gradient Any complete correct method using *their* perpendicular gradient to calculate straight line	**7**

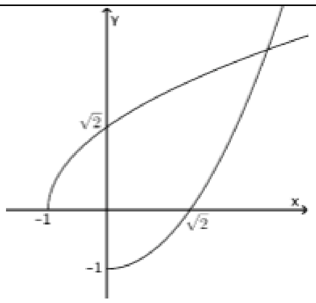
Question 12

The whole question was poorly done.

(b) Candidates struggled to find the domain.

(c) Candidates who did not know their shapes and used the table method struggled to sketch neat shapes and with that the domain was not taken into consideration. Candidates did not adhere to the instruction to indicate the intercept with the axes or did not use 3sf instead of the exact value.

(d) Candidates did not realize that differentiation was required and used $f^{-1}(x)$ instead.

12 (a)	$2k+2=0$ soi $k=-1$	M 1 A 1	 Condone $x=-1$ as a final answer.	
12 (b)	$x^2=2y+2$ $f^{-1}(x)=\frac{x^2-2}{2}$ Domain: $x\geq 0$	M 1 A 1 B 1	Or equivalent	
12 (c)	 	B 1 B 1 B 1 B 1	$f(x)$ Shape over the correct domain The correct 2 intercepts with axes and no others $f^{-1}(x)$ Shape over the correct domain, with a point of intersection in the first quadrant only. The correct 2 intercepts with axes and no others	
12 (d)	$f'(x)=\frac{1}{2}\times 2(2x+2)^{-\frac{1}{2}}$ $f'(7)=\frac{1}{4}$	M 1 A 1 A 1	For $k(2x+2)^{-\frac{1}{2}}$ All correct	**12**